NOTE ON EQUATION VII, SECTION 8 OF GAUSS'S THEORIA MOTUS.

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Put
$$\tan\frac{1}{2}a\tan\frac{1}{2}b\tan\frac{1}{2}c=1. \tag{1}$$

By trigonometry,
$$\tan^2\frac{1}{2}a=\frac{1-\cos a}{1+\cos a}, \text{ etc.}$$

Squaring, substituting, and clearing of fractions, (1) becomes
$$(1-\cos a)(1-\cos b)(1-\cos c)=(1+\cos a)(1+\cos b)(1+\cos c);$$
$$\therefore \cos a+\cos b+\cos c+\cos a\cos b\cos c=0;$$

whence
$$\cos a=\frac{-\cos b-\cos c}{1+\cos b\cos c}, \tag{2}$$

and, since $\sin a=\sqrt{1-\cos^2 a}$,
$$\sin a=\frac{\sin b\sin c}{1+\cos b\cos c}. \tag{3}$$

Put $a=v$, $b=90^\circ-\varphi$, $c=180^\circ-E$; and (1) becomes
$$\tan\frac{1}{2}v\tan(45^\circ-\frac{1}{2}\varphi)\cot\frac{1}{2}E=1,$$

which is equivalent to Gauss's equation VII. Also, (2) and (3) become
$$\cos v=\frac{\cos E-\sin\varphi}{1-\sin\varphi\cos E},$$
$$\sin v=\frac{\cos\varphi\sin E}{1-\sin\varphi\cos E};$$

and since, on account of the symmetry of (1), a, b, and c may be interchanged, we have also
$$\sin\varphi=\frac{\cos E-\cos v}{1-\cos v\cos E},$$
$$\cos\varphi=\frac{\sin v\sin E}{1-\cos v\cos E};$$

and
$$\cos E=\frac{\cos v+\sin\varphi}{1+\sin\varphi\cos v},$$
$$\sin E=\frac{\cos\varphi\sin v}{1+\sin\varphi\cos v}.$$